December 2004 Bicocca–FT–04–17

Covariant quantization of $N = \frac{1}{2}$ SYM theories and supergauge invariance

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ABSTRACT

So far, quantum properties of $N = 1/2$ nonanticommutative (NAC) super Yang–Mills theories have been investigated in the WZ gauge. The gauge independence of the results requires assuming that at the quantum level supergauge invariance is not broken by nonanticommutative geometry. In this paper we use an alternative approach which allows studying these theories in a manifestly gauge independent superspace setup. This is accomplished by generalizing the background field method to the NAC case, by moving to a momentum superspace where star products are treated as exponential factors and by developing momentum supergraph techniques. We compute the one–loop gauge effective action for NAC $U(\mathcal{N})$ gauge theories with matter in the adjoint representation. Despite the appearance of divergent contributions which break (super)gauge invariance, we prove that the effective action at this order is indeed invariant.

PACS: 03.70.+k, 11.15.-q, 11.10.-z, 11.30.Pb, 11.30.Rd

Keywords: Noncommutative geometry, $N = \frac{1}{2}$ Supersymmetry, Superspace, Yang–Mills theories, Background field method.

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1 Introduction

Euclidean superspace geometries with nonanticommutative (NAC) spinorial variables [1, 2] have been shown to arise as field theory limits of strings in flat spacetime when a self–dual graviphoton background is turned on [3, 4, 5, 6].

Field theories defined on such geometries describe the dynamics of bosonic and fermionic degrees of freedom which realize $(\frac{1}{2}, 0)$ supersymmetry [5]. They can be obtained by starting with ordinary extended $N = (1, 1)$ supersymmetry and explicitly breaking it, in a variety of ways, by turning on nontrivial anticommutation relations for spinorial coordinates (while in general supersymmetry is broken to $N = (\frac{1}{2}, 0)$, there are particular cases where $N = (\frac{1}{2}, 1)$ supersymmetry survives [7]).

Since in general supersymmetry plays an important role in guaranteing renormalization, in the presence of hard breaking it becomes of primary interest to investigate possible consequences for renormalizability. A first set of results in this direction has been obtained for the $N = (\frac{1}{2}, 0)$ WZ model in four dimensions. In particular, it has been shown [8, 9, 10, 11, 12, 13] that the model, initially defined by the ordinary WZ action where the products have been promoted to NAC star products [5] is renormalizable if extra F and F^2 interaction terms are added to the initial lagrangian, where F is the auxiliary field of the scalar multiplet.

The most interesting NAC theories to be investigated are supersymmetric $N = 1/2$ gauge theories. They are defined by the ordinary gauge actions where the products have been promoted to star products. $U(\mathcal{N})$, $N = 1/2$ super Yang–Mills is the field theory living on a set of $\mathcal{N}$ coincident D3–branes of a type IIB superstring theory compactified on a CY space, in the presence of a 4d self–dual graviphoton background [4, 6].

A general discussion of renormalizability for $N = 1/2$ super Yang–Mills theories based on dimensional analysis has been given [14] in the WZ gauge, while one–loop checks have been performed in [15, 16] still in components, in the WZ gauge. An advantage of working in the WZ gauge is that the expansion of the gauge superfield strengths in components gives rise to a finite number of terms also in the NAC case [5, 17]. In the ordinary anticommutative case, in the absence of anomalies, renormalizability in the WZ gauge guarantees renormalizability in any gauge since (super)gauge invariance is not spoiled by quantum corrections. However, in the presence of nonanticommutativity we do not have any *a priori* argument to guarantee that (super)gauge invariance will survive at the quantum level since supersymmetry breaking is realized by a star product which is defined in terms of ordinary non(super)covariant derivatives. [‡]. Therefore, quantum properties, renormalizability included, of NAC super Yang–Mills proved in the WZ gauge cannot

[‡]An analogous situation is present in super Yang–Mills theories defined on noncommutative geometries $[x^\mu, x^\nu] \neq 0$. There, it is well known [18, 19] that off–shell corrections to the effective action are not gauge invariant term by term, even if gauge invariance of the complete effective action does not seem to be affected [20].

be safely extended to any gauge without a deeper understanding of the relation among nonanticommutativity, susy breaking and (super)gauge invariance at the quantum level.

To investigate this subject we have been led to develop a different approach to the $N = 1/2$ super Yang–Mills theories which allows for a perturbative analysis directly in superspace without expanding in components in a particular gauge. This has also the advantage of allowing higher–loop calculations which in components are usually prohibited by technical difficulties. We work in $N = (\frac{1}{2}, 0)$ superspace keeping the star product implicit and performing Fourier transforms (FT) both in the bosonic and fermionic coordinates [§]. Under FT the star product is traded for exponential factors dependent on the spinorial momenta and the nonanticommuting matrix. As in the bosonic case, the diagrams can be classified into "planar" and "nonplanar", the nonplanar ones being the only diagrams having a nontrivial exponential factor dependent on internal and/or external spinorial momenta. In the ordinary case the supergraph techniques, in particular the D–algebra [22] which allows one to reduce supergraphs to ordinary loop momentum integrals, are easily translated in momentum superspace as a set of rules for the Fourier tranformed covariant derivatives $\widetilde{D}$. In the nonanticommuting case, instead, the presence of extra exponential factors from the star products affects in a nontrivial way the spinorial structure of the diagrams and, consequently, the $\widetilde{D}$–algebra. This is very different from the case of SYM theories defined on noncommuting superspaces where the noncommutation of the bosonic coordinates does not modify the spinorial nature of the supergraphs and D–algebra (or $\widetilde{D}$–algebra) can be performed with the standard rules of the ordinary commutative case [19].

In the $N = (\frac{1}{2}, 0)$ superspace setting we use the general procedure described above to study quantum properties of $U(\mathcal{N})$ super Yang–Mills theories with (anti)chiral matter in the adjoint representation of the gauge group [¶]. In order to carry out the calculations by dealing efficiently with the classical gauge covariance we generalize the background field method [23, 22] to the nonanticommutative case. The generalization is not straightforward, primarily due to two reasons: The change of the hermitian conjugation rules in the classical action (the NAC superspace has euclidean signature [2, 5]) and the lack of basic identities involving covariant derivatives which in the NAC case are spoiled by the noncommutativity of the star product. However, we show that a modified version of the method exists which allows for a manifestly covariant quantization of gauge theories, at least at one–loop.

Armed with these techniques we compute one–loop divergent contributions to the gauge effective action. In the presence of spinorial phases coming from the star products the divergent nature of the diagrams changes in a nontrivial way. As a consequence, we find divergent contributions to the two, three and four–point functions, in contradistinction to the ordinary case where, in the background field method approach, only the

[§]Similar techniques have been used in [13, 21].

[¶]We note that in the presence of star product the group $SU(\mathcal{N})$ is not closed and we are forced to consider $U(\mathcal{N})$.

two–point function is divergent [23, 22].

The new divergent terms, proportional to the NAC parameter, are *not* (super)gauge invariant on their own. However, we prove that at the level of effective action highly nontrivial cancellations among nonvanishing gauge variations occur leading to a one–loop effective action for the gauge fields which is *supergauge invariant*. Therefore, supergauge invariance seems to be maintained at the quantum level despite the presence of the star product which is not manifestly supergauge invariant. A similar discussion can be found in a paper [16] which appeared few days ago, when our work was almost finished. There the authors show that, working in components in the WZ gauge, a one–loop divergent field redefinition of the gaugino field is needed in order to restore gauge invariance at one–loop. Our results prove that in the gauge of superspace no field redefinition is required and the effective action turns out to be not only gauge, but even supergauge invariant at one–loop.

The paper is organized as follows: In Section 2 we briefly review the $N = (\frac{1}{2}, 0)$ NAC superspace [5] and supersymmetric gauge theories defined on it, and formulate the background field method in the presence of nonaticommutativity. The Fourier transform to momentum superspace and the corresponding supergraph techniques are then described in Section 4. In Section 5 we concentrate on one–loop calculations of two, three and four–point functions with external vector lines for the $U(\mathcal{N})$ SYM with matter in the adjoint. In Section 6 we collect all the results and prove the supergauge invariance of the one–loop divergent part of the effective action. The last Section is then devoted to some conclusions. In this paper we basically list the main results referring to a future publication [24] for details and an extended analysis.

2 SYM theories in $N = 1/2$ superspace and background field method

Nonanticommutative $N = (\frac{1}{2}, 0)$ superspace can be defined as a truncation of euclidean $N = (1, 1)$ superspace endowed with nonstandard hermitian conjugation rules on the spinorial variables [8, 5, 7]. It is described by the set of coordinates $(x^{\alpha\dot{\alpha}}, \theta^\alpha, \overline{\theta}^{\dot{\alpha}})$, $(\theta^\alpha)^\dagger = i\theta_\alpha$, $(\overline{\theta}^{\dot{\alpha}})^\dagger = i\overline{\theta}_{\dot{\alpha}}$, satisfying

$$\{\theta^\alpha, \theta^\beta\} = 2\mathcal{F}^{\alpha\beta} \qquad \text{the rest} = 0 \tag{2.1}$$

where $\mathcal{F}^{\alpha\beta}$ is a 2×2 symmetric, constant matrix. This algebra is consistent only in euclidean signature where the chiral and antichiral sectors are totally independent and not related by complex conjugation. The euclidean Lorentz group $SO(4) = SU_L(2) \times SU_R(2)$ is broken to $SU_R(2)$ by (2.1).

We work in chiral representation [22] for supercharges and covariant derivatives

$$\begin{aligned} \overline{Q}_{\dot\alpha} &= i(\overline{\partial}_{\dot\alpha} - i\theta^\alpha \partial_{\alpha\dot\alpha}) \quad , \quad Q_\alpha = i\partial_\alpha \\ \overline{D}_{\dot\alpha} &= \overline{\partial}_{\dot\alpha} \qquad\qquad\qquad , \quad D_\alpha = \partial_\alpha + i\overline{\theta}^{\dot\alpha} \partial_{\alpha\dot\alpha} \end{aligned} \tag{2.2}$$

While the algebra of covariant derivatives is not modified, the algebra of supercharges receives an extra contribution from (2.1) and the supersymmetry in the antichiral sector is explicitly broken [5].

The NAC geometry (2.1) can be realized on the class of smooth superfunctions of $(x^{\alpha\dot\alpha}, \theta^\alpha, \overline{\theta}^{\dot\alpha})$, by introducing the nonanticommutative (but associative) star product

$$\begin{aligned} \phi * \psi &\equiv \phi e^{-\overleftarrow{\partial}_\alpha \mathcal{F}^{\alpha\beta} \overrightarrow{\partial}_\beta} \psi \\ &= \phi\psi - \phi \overleftarrow{\partial}_\alpha \mathcal{F}^{\alpha\beta} \overrightarrow{\partial}_\beta \psi + \frac{1}{2} \phi \overleftarrow{\partial}_\alpha \overleftarrow{\partial}_\gamma \mathcal{F}^{\alpha\beta} \mathcal{F}^{\gamma\delta} \overrightarrow{\partial}_\delta \overrightarrow{\partial}_\beta \psi \\ &= \phi\psi - \phi \overleftarrow{\partial}_\alpha \mathcal{F}^{\alpha\beta} \overrightarrow{\partial}_\beta \psi - \frac{1}{2} \mathcal{F}^2 \partial^2 \phi \, \partial^2 \psi \end{aligned} \tag{2.3}$$

where we have defined $\mathcal{F}^2 \equiv \mathcal{F}^{\alpha\beta}\mathcal{F}_{\alpha\beta}$. The covariant derivatives (2.2) are still derivations for this product. Therefore, the class of (anti)chiral superfields defined by the constraints $\overline{D}_{\dot\alpha} * \Phi = D_\alpha * \overline{\Phi} = 0$ are closed.

We now define supersymmetric gauge theories on $N = 1/2$ superspace. Since in the presence of non(anti)commutativity also the $U_*(1)$ gauge theory becomes nonabelian the relations we are going to introduce hold nontrivially for any gauge group, $U_*(1)$ included.

Gauge fields and field strengths together with their superpartners can be organized into superfields, all expressed in terms of a prepotential V which is a scalar superfield in the adjoint representation of the gauge group ($V \equiv V_a T^a$, T^a being the group generators).

The supergauge transformations are given in terms of two independent chiral and antichiral parameter superfields $\Lambda, \overline{\Lambda}$

$$e_*^V \to e_*^{V'} = e_*^{i\overline{\Lambda}} * e_*^V * e_*^{-i\Lambda} \tag{2.4}$$

The corresponding covariant derivatives (in gauge chiral representation) are given by

$$\nabla_A \equiv (\nabla_\alpha, \nabla_{\dot\alpha}, \nabla_{\alpha\dot\alpha}) \;=\; (e_*^{-V} * D_\alpha \, e_*^V \;,\; \overline{D}_{\dot\alpha} \;,\; -i\{\nabla_\alpha, \nabla_{\dot\alpha}\}_*) \tag{2.5}$$

whereas in gauge antichiral representation they are defined as

$$\overline{\nabla}_A \equiv (\overline{\nabla}_\alpha, \overline{\nabla}_{\dot\alpha}, \overline{\nabla}_{\alpha\dot\alpha}) \;=\; (D_\alpha \;,\; e_*^V * \overline{D}_{\dot\alpha} \, e_*^{-V} \;,\; -i\{\overline{\nabla}_\alpha, \overline{\nabla}_{\dot\alpha}\}_*) \tag{2.6}$$

satisfying $\overline{\nabla}_A = e_*^V * \nabla_A * e_*^{-V}$.

They can be expressed in terms of ordinary supercovariant derivatives $D_A, \overline{D}_A$ and a set of connections, as $\nabla_A \equiv D_A - i\Gamma_A$ or $\overline{\nabla}_A \equiv \overline{D}_A - i\overline{\Gamma}_A$. Nontrivial connections are then

$$\Gamma_\alpha = i e_*^{-V} * D_\alpha \, e_*^V \qquad , \qquad \Gamma_{\alpha\dot{\alpha}} = -i\overline{D}_{\dot{\alpha}}\Gamma_\alpha \tag{2.7}$$

or

$$\overline{\Gamma}_{\dot{\alpha}} = i e_*^V * \overline{D}_{\dot{\alpha}} \, e_*^{-V} \qquad , \qquad \overline{\Gamma}_{\alpha\dot{\alpha}} = -i D_\alpha \overline{\Gamma}_{\dot{\alpha}} \tag{2.8}$$

The field strengths are defined as $*$–commutators of supergauge covariant derivatives

$$W_\alpha = -\frac{1}{2}[\nabla^{\dot{\alpha}}, \nabla_{\alpha\dot{\alpha}}]_* \qquad , \qquad W_{\dot{\alpha}} = -\frac{1}{2}[\nabla^{\alpha}, \nabla_{\alpha\dot{\alpha}}]_* \tag{2.9}$$

or

$$\overline{W}_\alpha = -\frac{1}{2}[\overline{\nabla}^{\dot{\alpha}}, \overline{\nabla}_{\alpha\dot{\alpha}}]_* \qquad , \qquad \overline{W}_{\dot{\alpha}} = -\frac{1}{2}[\overline{\nabla}^{\alpha}, \overline{\nabla}_{\alpha\dot{\alpha}}]_* \tag{2.10}$$

and satisfy the Bianchi's identities $\nabla^\alpha * W_\alpha + \nabla^{\dot{\alpha}} * W_{\dot{\alpha}} = 0$ or $\overline{\nabla}^\alpha * \overline{W}_\alpha + \overline{\nabla}^{\dot{\alpha}} * \overline{W}_{\dot{\alpha}} = 0$.

In the presence of chiral matter in the adjoint representation of the gauge group $\mathcal{G}$ the SYM action is

$$\begin{aligned} S \;=\; & \int d^4x d^4\theta \; \mathrm{Tr}(e_*^{-V} * \overline{\Phi} * e_*^V * \Phi) + \frac{1}{2g^2}\int d^4x d^2\theta \; \mathrm{Tr}(W^\alpha W_\alpha) \\ & -\frac{1}{2} m \int d^4x d^2\theta \; \Phi^2 - \frac{1}{2}\overline{m} \int d^4x d^2\overline{\theta} \; \overline{\Phi}^2 \end{aligned} \tag{2.11}$$

where all the superfield can be consistently taken to be real (we are in Euclidean superspace). In what follows we work with $\mathcal{G} = U(\mathcal{N})$.

We now generalize the background field method [23, 22] to the case of NAC super Yang–Mills theories with chiral matter in a real representation of the gauge group. We perform the nonlinear splitting $e_*^V \to e_*^\Omega * e_*^V$ where Ω is the background prepotential, and write the covariant derivatives (in gauge-chiral representation) as

$$\nabla_\alpha = e_*^{-V} * \boldsymbol{\nabla}_\alpha * e_*^V \qquad , \qquad \nabla_{\dot{\alpha}} \equiv \boldsymbol{\nabla}_{\dot{\alpha}} = \overline{D}_{\dot{\alpha}} \tag{2.12}$$

with similar expressions for $(\overline{\nabla}_\alpha, \overline{\nabla}_{\dot{\alpha}})$. These derivatives transform covariantly with respect to two types of gauge transformations: quantum transformations

$$\begin{aligned} & e_*^V \to e_*^{i\overline{\Lambda}} * e_*^V * e_*^{-i\Lambda} \\ \nabla_A \to e_*^{i\Lambda} * \nabla_A * e_*^{-i\Lambda} \qquad & , \qquad \boldsymbol{\nabla}_A \to \boldsymbol{\nabla}_A \\ \overline{\nabla}_A \to e_*^{i\overline{\Lambda}} * \overline{\nabla}_A * e_*^{-i\overline{\Lambda}} \qquad & , \qquad \overline{\boldsymbol{\nabla}}_A \to \overline{\boldsymbol{\nabla}}_A \end{aligned} \tag{2.13}$$

with background covariantly (anti)chiral parameters, $\boldsymbol{\nabla}_\alpha * \overline{\Lambda} = \boldsymbol{\nabla}_{\dot{\alpha}}\Lambda = 0$, and background transformations

$$\begin{aligned} & e_*^V \to e_*^{iK} * e_*^V * e_*^{-iK} \\ \nabla_A \to e_*^{iK} * \nabla_A * e_*^{-iK} \qquad & , \qquad \boldsymbol{\nabla}_A \to e_*^{iK} * \boldsymbol{\nabla}_A * e_*^{-iK} \\ \overline{\nabla}_A \to e_*^{iK} * \overline{\nabla}_A * e_*^{-iK} \qquad & , \qquad \overline{\boldsymbol{\nabla}}_A \to e_*^{iK} * \overline{\boldsymbol{\nabla}}_A * e_*^{-iK} \end{aligned} \tag{2.14}$$

with real parameter K.

Full covariantly (anti)chiral superfields $\nabla_{\dot\alpha}\Phi = \nabla_\alpha * \overline{\Phi} = 0$ are expressed in terms of background (anti)chiral superfields as $\Phi = \Phi_0$, $\overline{\Phi} = \overline{\Phi}_0 * e_*^V$, $\boldsymbol{\nabla}_{\dot\alpha} * \Phi_0 = 0$, $\boldsymbol{\nabla}_\alpha * \overline{\Phi}_0 = 0$ and then linearly split into a background and a quantum part. Under quantum transformations the fields transform as $\Phi' = e_*^{i\Lambda} * \Phi$, $\overline{\Phi}' = \overline{\Phi} * e_*^{-i\overline{\Lambda}}$, whereas under background transformations they transform as $\Phi' = e_*^{iK} * \Phi$, $\overline{\Phi}' = \overline{\Phi} * e_*^{-iK}$.

The classical action (2.11) is invariant under the transformations (2.13, 2.14). Background field quantization consists in performing gauge–fixing which explicitly breaks the (2.13) gauge invariance while preserving manifest invariance under (2.14). The procedure follows closely the ordinary one [22] by simply replacing products with star products. It leads to a gauge–fixed action $S_{tot} = S_{inv} + S_{GF} + S_{gh}$ where S_{gh} is given in terms of background covariantly (anti)chiral FP and NK ghost superfields and the quadratic part reads

$$S_{gh} = \int d^4x d^4\theta \left[\overline{c}'c - c'\overline{c} + \overline{b}b\right] \tag{2.15}$$

From the rest of the action we read the V propagator which in the Feynman gauge is

$$\langle V_a(z) V_b(z')\rangle = g^2 \frac{\delta_{ab}}{\Box_0}\, \delta^{(4)}(\theta - \theta') \tag{2.16}$$

and the pure gauge interaction terms useful for one–loop calculations

$$\begin{aligned} -\frac{1}{2g^2}\int d^4x d^4\theta \; \mathrm{Tr} V\Big[&- i[\Gamma^{\alpha\dot\alpha}, \partial_{\alpha\dot\alpha} V]_* - i\{W^\alpha, D_\alpha V\}_* - i\{W^{\dot\alpha}, \overline{D}_{\dot\alpha} V\}_* \\ &- \frac{1}{2}[\Gamma^{\alpha\dot\alpha}, [\Gamma_{\alpha\dot\alpha}, V]]_* - \{W^\alpha, [\Gamma_\alpha, V]\}_* - \{W^{\dot\alpha}, [\Gamma_{\dot\alpha}, V]\}_*\Big] \end{aligned} \tag{2.17}$$

We now turn to the action for matter in a real representation of the gauge group. In particular, ghosts fall in this category so the following procedure can be applied to the action (2.15).

In the ordinary case, in terms of covariantly chiral and antichiral superfields Φ and $\overline{\Phi}$ (related by complex conjugation)

$$S = \int d^4x d^4\theta \; \overline{\Phi}\Phi \tag{2.18}$$

The corresponding equations of motion

$$\mathcal{O}\begin{pmatrix} \Phi \\ \overline{\Phi} \end{pmatrix} = 0 \qquad\qquad \mathcal{O} = \begin{pmatrix} 0 & \overline{D}^2 \\ \nabla^2 & 0 \end{pmatrix} \tag{2.19}$$

can be formally derived from the functional determinant

$$\Delta \;=\; \int \mathcal{D}\Psi e^{\overline{\Psi}\mathcal{O}\Psi} \sim (\det \mathcal{O})^{-\frac{1}{2}} \tag{2.20}$$

where Ψ is the column vector $\begin{pmatrix} \Phi \\ \overline{\Phi} \end{pmatrix}$. If we perform the change of variables $\Psi = \sqrt{\mathcal{O}}\Psi'$, whose jacobian is det$\sqrt{\mathcal{O}} = \Delta^{-\frac{1}{2}}$, we can write

$$\Delta = \int \mathcal{D}\Psi' \; \Delta^{-1} \; e^{\overline{\Psi}' \mathcal{O}^2 \Psi'} \tag{2.21}$$

or equivalently

$$\Delta^2 = \int \mathcal{D}\Psi e^{\overline{\Psi}\mathcal{O}^2\Psi} \tag{2.22}$$

where

$$\mathcal{O}^2 = \begin{pmatrix} \overline{D}^2\nabla^2 & 0 \\ 0 & \nabla^2\overline{D}^2 \end{pmatrix} \tag{2.23}$$

is a diagonal matrix. Therefore, defining the actions

$$\begin{aligned} S' &= \frac{1}{2}\int d^4x d^4\theta \; \Phi\nabla^2\Phi \\ \overline{S}' &= \frac{1}{2}\int d^4x d^4\theta \; \overline{\Phi}\overline{D}^2\overline{\Phi} \end{aligned} \tag{2.24}$$

it is easy to see that the following chain of identities holds [22]

$$\Delta^2 \;=\; \int \mathcal{D}\Phi\mathcal{D}\overline{\Phi} \; e^{S'+\overline{S}'} \;=\; \left| \int \mathcal{D}\Phi e^{S'} \right|^2 \;=\; \left(\int \mathcal{D}\Phi e^{S'} \right)^2 \tag{2.25}$$

where we have used $\overline{S}' = (S')^\dagger$ and the fact that they both contribute in the same way to Δ [22]. Therefore, when Δ is real, we can identify the original Δ with $\int \mathcal{D}\Phi e^{S'}$ and derive from here the Feynman rules [22].

We now extend the previous derivation to the case of NAC euclidean superspace where all the h.c. relations are relaxed and Φ, $\overline{\Phi}$ are two independent but *real* superfields. The matter action is still given by (2.18) and we can still define Δ as in (2.21). Therefore, we write

$$\Delta_* = \int \mathcal{D}\Psi \; e^{\Psi^T * \mathcal{O} * \Psi} \sim (\det(\mathbf{O}))^{-\frac{1}{2}} \tag{2.26}$$

We can then proceed as before and square the functional integral to obtain

$$\Delta_*^2 = \int \mathcal{D}\Psi^T e^{\Psi * \mathbf{O}^2 * \Psi} \tag{2.27}$$

where $\mathbf{O}^2$ is given in (2.23) with the products promoted to star products. Now, if we introduce

$$\Delta_1 = \int \mathcal{D}\Phi e^{S'} \qquad , \qquad \Delta_2 = \int \mathcal{D}\overline{\Phi} e^{\overline{S}'} \tag{2.28}$$

with S', $\overline{S}'$ still given in (2.24) we can finally write

$$\Delta_*^2 = \Delta_1\Delta_2 \tag{2.29}$$

In contradistinction to the ordinary case, now $\overline{S}' \neq (S')^\dagger$. Moreover, the star product, when expanded, could in principle generate different terms in the two actions. Therefore, the chain of identities (2.25) is not immediately generalizable to the NAC case and we cannot identify $\Delta_* = \Delta_1$.

However, given the equality (2.29) the Feynman rules for Δ_* can be still inferred from $\Delta_{1,2}$ order by order. In fact, we consider $\Delta_*, \Delta_1, \Delta_2$ as functions of the coupling constant g and perform a perturbative expansion

$$\begin{aligned} \Delta_*[g] \quad &= \Delta_*[0] + g^2\Delta'_*[0] + \cdots \\ \Delta_1[g] \quad &= \Delta_1[0] + g^2\Delta'_1[0] + \cdots \\ \Delta_2[g] \quad &= \Delta_2[0] + g^2\Delta'_2[0] + \cdots \end{aligned} \tag{2.30}$$

Normalizing the functionals as $\Delta_*[0] = \Delta_1[0] = \Delta_2[0]$ and expanding the identity (2.29) in powers of g we obtain

$$\Delta_*^2 = (1 + g^2\Delta'_*[0] + \cdots)^2 = (1 + g^2\Delta'_1[0] + \cdots)(1 + g^2\Delta'_2[0] + \cdots) \tag{2.31}$$

In particular, since we are interested in computing one–loop contributions to the effective action at order g^2 we find

$$2\Delta'_*[0] = \Delta'_1[0] + \Delta'_2[0] \tag{2.32}$$

Therefore at one loop Δ_* is given by the sum of the contributions from S' and $\overline{S}'$.

Following closely the ordinary case [22] we derive the Feynman rules from S' and $\overline{S}'$ by first extracting the quadratic part of the actions and then reading the vertices from the rest.

Since for the chiral action the identities involving covariant derivatives are formally the same except for the products which are now star products, the procedure to obtain the analytic expressions associated to the vertices is formally the same. We then refer the reader to Ref. [22] for details while reporting here only the final rules:

- Propagator

$$\langle \Phi(z)\Phi(z')\rangle = -\frac{1}{\Box_0}\delta^{(4)}(\theta - \theta') \tag{2.33}$$

- Chiral vertices: at one loop the prescription requires associating with one vertex

$$\frac{1}{2}\overline{D}^2(\nabla^2 - D^2) \tag{2.34}$$

  and with the other vertices

$$\frac{1}{2}(\Box_+ - \Box_0) \tag{2.35}$$

  where $\Box_+ = \Box_{cov} - iW^\alpha * \nabla_\alpha - \frac{i}{2}(\nabla^\alpha * W_\alpha)$, $\qquad \Box_{cov} = \frac{1}{2}\nabla^{\alpha\dot{\alpha}} * \nabla_{\alpha\dot{\alpha}}$.

The procedure can be easily extended to the case of massive chirals by simply promoting the propagators (2.33) to massive propagators $-1/(\Box_0 - m\overline{m})$. We also note that these rules are strictly one-loop rules. At higher orders there are no difficulties and ordinary rules apply, as described in [22] with obvious modifications required by noncommutativity.

We can write down a formal effective interaction lagrangian that corresponds to the one-loop rules above. In the case of massive matter (chirals with mass m and antichirals with $\overline{m}$) in the adjoint representation of the gauge group, it is given by (from now on we avoid indicating star products when no confusion arises)

$$S_0 + S_1 + S_2 \equiv \int d^4x d^4\theta \ \mathrm{Tr} \left\{ \overline{\psi}(\Box_0 - m\overline{m})\psi + \frac{1}{2} \left[\overline{\psi} \ \overline{D}^2 (\nabla^2 - D^2)\psi \ + \ \overline{\psi}(\Box_+ - \Box_0)\psi \right] \right\} \tag{2.36}$$

where $\psi, \overline{\psi}$ are *quantum unconstrained* superfields and the first vertex has to appear once and only once in any one–loop diagram.
By writing everything explicitly in terms of connections and field strengths and performing some integrations by parts it can be rewritten as (we neglect terms with lower powers of $\overline{D}$ which would not contribute in one–loop calculations)

$$\begin{aligned} S_1 &= \int d^4x d^4\theta \ \mathrm{Tr} \left\{ \left(\frac{i}{4}\Gamma^\alpha [\overline{D}^2\psi, D_\alpha\overline{\psi}] - \frac{i}{4}\Gamma^\alpha [\overline{D}^2 D_\alpha\psi, \overline{\psi}] \right) + \left(-\frac{1}{4}\overline{\psi}\{\Gamma^\alpha [\Gamma_\alpha, \overline{D}^2\psi]\} \right) \right\} \\ &\equiv S_1 + S_1' \end{aligned} \tag{2.37}$$

$$\begin{aligned} S_2 &= \int d^4x d^4\theta \ \mathrm{Tr} \left\{ \left(\frac{i}{4}\Gamma^{\alpha\dot{\alpha}} [\psi, \partial_{\alpha\dot{\alpha}}\overline{\psi}] - \frac{i}{4}\Gamma^{\alpha\dot{\alpha}} [\partial_{\alpha\dot{\alpha}}\psi, \overline{\psi}] \right) + \left(\frac{i}{4}W^\alpha [\psi, D_\alpha\overline{\psi}] - \frac{i}{4}W^\alpha [D_\alpha\psi, \overline{\psi}] \right) \right. \\ &\qquad \left. + \ \left(\frac{1}{4}[\Gamma^\alpha, \psi][W_\alpha, \overline{\psi}] + \frac{1}{4}[W^\alpha, \psi][\Gamma_\alpha, \overline{\psi}] \right) + \left(\frac{1}{4}[\Gamma^{\alpha\dot{\alpha}}, \psi][\Gamma_{\alpha\dot{\alpha}}, \overline{\psi}] \right) \right\} \\ &\equiv S_2 + S_2' + S_2'' + S_2''' \end{aligned} \tag{2.38}$$

To extract the Feynman rules for the antichiral sector, we have to go carefully through the whole procedure since some of the identities used in the ordinary case do not hold anymore because of the noncommutative product. We find convenient to express the action $\overline{S}'$ in terms of ordinary (not covariantly) antichiral superfields. Using cyclicity under trace and $d^4\theta$ integration we find

$$\overline{S}' = \frac{1}{2} \int d^4x d^4\theta \ \mathrm{Tr}(\overline{\Phi} e^V \overline{D}^2 e^{-V} \overline{\Phi}) = \frac{1}{2} \int d^4x d^2\overline{\theta} \ \mathrm{Tr}(\overline{\Phi} D^2 \overline{\nabla}^2 \overline{\Phi}) \tag{2.39}$$

where $\overline{\nabla}^2 = e_*^V * \overline{D}^2 e_*^{-V}$. Using covariant derivatives in the antichiral representation we can formally follow the same procedure of the chiral sector by changing bar quantities into unbar ones, and viceversa. Therefore, the Feynman rules are:

- Propagator

$$\langle \overline{\Phi}(z) \overline{\Phi}(z') \rangle = -\frac{1}{\Box_0} \delta^{(4)}(\theta - \theta') \tag{2.40}$$

- one vertex: $\frac{1}{2}D^2(\overline{\nabla}^2 - \overline{D}^2)$
- other vertices: $\frac{1}{2}(\overline{\Box}_+ - \Box_0)$
  with $\overline{\Box}_+ = \overline{\Box}_{cov} - i\overline{W}^{\dot\alpha} * \overline{\nabla}_{\dot\alpha} - \frac{i}{2}(\overline{\nabla}^{\dot\alpha} * \overline{W}_{\dot\alpha})$ (here $\overline{W}_{\dot\alpha}$ is the antichiral field strength and $\overline{\Box}_{cov} = \frac{1}{2}\overline{\nabla}^{\alpha\dot\alpha} * \overline{\nabla}_{\alpha\dot\alpha}$).

Again, it is convenient to introduce an effective action in terms of quantum unconstrained superfields ξ and $\overline{\xi}$

$$\overline{S}_0 + \overline{S}_1 + \overline{S}_2 \equiv \int d^4x d^4\theta \ \mathrm{Tr} \left\{ \overline{\xi}(\Box_0 - m\overline{m})\xi + \frac{1}{2}\left[\overline{\xi} D^2(\overline{\nabla}^2 - \overline{D}^2)\xi \ + \ \overline{\xi}(\overline{\Box}_+ - \Box_0)\xi \right] \right\} \tag{2.41}$$

In terms of connections and field strengths it can be rewritten as

$$\begin{aligned} \overline{S}_1 &= \int d^4x d^4\theta \ \mathrm{Tr} \left\{ \left(\frac{i}{4}\overline{\Gamma}^{\dot\alpha}[\xi, \overline{D}_{\dot\alpha} D^2 \overline{\xi}] - \frac{i}{4}\overline{\Gamma}^{\dot\alpha}[\overline{D}_{\dot\alpha}\xi, D^2\overline{\xi}] \right) + \left(-\frac{1}{4}\overline{\xi}\{\overline{\Gamma}^{\dot\alpha}[\overline{\Gamma}_{\dot\alpha}, D^2\xi]\} \right) \right\} \\ &\equiv \overline{S}_1 + \overline{S}_1' \end{aligned} \tag{2.42}$$

$$\begin{aligned} \overline{S}_2 &= \int d^4x d^4\theta \ \mathrm{Tr} \left\{ \left(\frac{i}{4}\overline{\Gamma}^{\alpha\dot\alpha}[\xi, \partial_{\alpha\dot\alpha}\overline{\xi}] - \frac{i}{4}\overline{\Gamma}^{\alpha\dot\alpha}[\partial_{\alpha\dot\alpha}\xi, \overline{\xi}] \right) + \left(\frac{i}{4}\overline{W}^{\dot\alpha}[\xi, \overline{D}_{\dot\alpha}\overline{\xi}] - \frac{i}{4}\overline{W}^{\dot\alpha}[\overline{D}_{\dot\alpha}\xi, \overline{\xi}] \right) \right. \\ &\qquad \left. + \ \left(\frac{1}{4}[\overline{\Gamma}^{\dot\alpha}, \xi][\overline{W}_{\dot\alpha}, \overline{\xi}] + \frac{1}{4}[\overline{W}^{\dot\alpha}, \xi][\overline{\Gamma}_{\dot\alpha}, \overline{\xi}] \right) + \left(\frac{1}{4}[\overline{\Gamma}^{\alpha\dot\alpha}, \xi][\overline{\Gamma}_{\alpha\dot\alpha}, \overline{\xi}] \right) \right\} \\ &\equiv \overline{S}_2 + \overline{S}_2' + \overline{S}_2'' + \overline{S}_2''' \end{aligned} \tag{2.43}$$

3 Supergraph techniques in momentum frame

As in the ordinary supersymmetric theories, it is convenient to develop a procedure which allows one to study NAC field theories perturbatively without going to components, in particular without expanding necessarily the star product. As already indicated in the Introduction, such a procedure is unavoidable in dealing with NAC super Yang–Mills theories when the relation between nonaticommutativity and gauge invariance is under study.

In this section we describe the main lines of our approach. Details will be reported in a future publication [24].

Given a generic superfunction Φ living on the NAC superspace we move to *momentum superspace* by Fourier transforming both the bosonic and fermionic coordinates according to the prescription

$$\widetilde{\Phi}(p, \pi, \overline{\pi}) = \int d^4x d^2\theta d^2\overline{\theta} \ e^{ipx + i\pi\theta + i\overline{\pi}\overline{\theta}} \ \Phi(x, \theta, \overline{\theta}) \tag{3.1}$$

In momentum superspace the star product is traded for an exponential factor dependent on the spinorial momentum variables

$$\Phi(x,\theta,\overline{\theta}) * \Psi(x,\theta,\overline{\theta}) \longrightarrow e^{\pi\wedge\pi'}\ \widetilde{\Phi}(p,\pi,\overline{\pi})\widetilde{\Psi}(p',\pi',\overline{\pi}') \tag{3.2}$$

where we have defined $\pi \wedge \pi' \equiv \pi_\alpha \mathcal{F}^{\alpha\beta}\pi'_\beta$.

We then develop perturbative techniques in momentum superspace. In the ordinary anticommutative case this amounts to translating Feynman rules for propagators and vertices to momentum language. Spinorial D derivatives become $\tilde{D}$ derivatives according to the relations

$$\begin{aligned} D_\alpha = \partial_\alpha + i\overline{\theta}^{\dot{\alpha}}\partial_{\alpha\dot{\alpha}} \quad &\rightarrow \quad \widetilde{D}_\alpha = -i\pi_\alpha - i\overline{\eth}^{\dot{\alpha}} p_{\alpha\dot{\alpha}} \\ \overline{D}_{\dot{\alpha}} = \overline{\partial}_{\dot{\alpha}} \quad &\rightarrow \quad \widetilde{\overline{D}}_{\dot{\alpha}} = -i\overline{\pi}_{\dot{\alpha}} \end{aligned} \tag{3.3}$$

where we have indicated $\overline{\eth}_{\dot{\alpha}} = \frac{\partial}{\partial\overline{\pi}^{\dot{\alpha}}}$. The ordinary D–algebra which allows reducing a supergraph to an ordinary momentum diagram gets translated into a $\tilde{D}$–algebra in a straightforward way. In particular, while in configuration superspace the general rule to get a notrivial contribution from a given supergraph is to perform D–algebra until we are left with a factor $D^2\overline{D}^2$ for each loop, in momentum superspace it gets translated into the requirement to perform $\tilde{D}$–algebra until one ends up with a factor $\pi^2\overline{\pi}^2$ for each loop, where $(\pi,\overline{\pi})$ are the loop spinorial momenta.

In the NAC case we again translate the Feynman rules to momentum superspace. However, relevant changes occur due to the appearance of exponential factors at the vertices. Thus, given a local cubic vertex of the form $\int A * B * C$ the corresponding expression in momentum superspace becomes (we indicate $\Pi \equiv (p,\pi,\overline{\pi})$)

$$e^{\pi_1\wedge\pi_2}\ \tilde{A}(\Pi_1)\tilde{B}(\Pi_2)\tilde{C}(\Pi_3)\ \delta^{(8)}(\Pi_1+\Pi_2+\Pi_3) \tag{3.4}$$

More generally a local n–point vertex gives

$$\prod_{i<j}^{n} e^{\pi_i\wedge\pi_j}\ \tilde{A}_1(\Pi_1)\cdots\tilde{A}_n(\Pi_n)\ \delta^{(8)}(\sum_i \Pi_i) \tag{3.5}$$

When contracting quantum lines coming out from the vertices to build Feynman diagrams, different ways of performing contractions lead to different configurations of exponential factors. Due to spinorial momentum conservation at each vertex the diagrams can be classified into *planar* diagrams characterized by exponentials depending only on the external momenta and *nonplanar* ones which have a nontrivial exponential dependence on the loop momenta. This pattern resembles closely what happens in the case of bosonic noncommutative theories [25, 26] except for the exponential factors which in that case are actual phase factors. However, this does not prevent us from using the same prescriptions [25, 26] to determine the overall exponential factor associated with a given diagram.

Once the exponential factor and the structure of the $\tilde{D}$ derivatives associated to a given diagram have been established we proceed by performing $\tilde{D}$–algebra. This amounts to using suitable identities to reduce the number of spinorial derivatives, expanding the exponential factors as

$$e^{\pi_1 \wedge \pi_2} = 1 + \pi_1^\alpha \mathcal{F}_{\alpha\beta} \pi_2^\beta - \frac{1}{2} \pi_1^2 \mathcal{F}^2 \pi_2^2 \tag{3.6}$$

and selecting those configurations of spinorial momenta which have a factor $\pi^2 \overline{\pi}^2$ for each loop. We note that while $\overline{\pi}^2$ factors only come from $\tilde{\overline{D}}$ derivatives associated to the vertices as in the ordinary case, π^2 factors can also come from the expansion (3.6), giving extra nonvanishing contributions to a given diagram proportional to the nonanticommutation parameter $\mathcal{F}$. This is the way nonanticommutativity enters the calculations in our approach.

Finally, once $\tilde{D}$–algebra has been performed, we are left with ordinary momentum loop integrals. We evaluate them in dimensional regularization ($n = 4 - 2\epsilon$) and in the G–scheme [27] in order to avoid dealing with irrelevant constants coming from the expansion of gamma functions.

Before closing this section we give the Feynman rules in momentum superspace needed for calculations at one–loop in the background field approach.

The propagators for the gauge superfield and massive matter are (see eqs. (2.16, 2.36))

$$\begin{aligned}
\langle \tilde{V}^a(\Pi) \tilde{V}^b(\Pi') \rangle &= -g^2 \frac{\delta^{ab}}{p^2} \; \delta^{(8)}(\Pi + \Pi') \\
\langle \tilde{\psi}(\Pi) \tilde{\overline{\psi}}(\Pi') \rangle &= \frac{1}{p^2 + m\overline{m}} \; \delta^{(8)}(\Pi + \Pi')
\end{aligned} \tag{3.7}$$

where we take the momenta always entering the vertex.

The vertices quadratic in the quantum V superfield are given in (2.17). The only term which effectively contributes will be the one which contains a $\overline{D}$. Performing Fourier transform as described above the corresponding vertex gets the structure (we indicate $\Pi_i \equiv (p_i, \pi_i, \overline{\pi}_i)))$

$$\begin{aligned}
&\frac{1}{2g^2} \int d^8\Pi_1 d^8\Pi_2 d^8\Pi_3 \; \delta^{(8)}(\Pi_1 + \Pi_2 + \Pi_3) \; \widetilde{W}^{\dot{\alpha}\, a}(\Pi_1) \overline{\pi}_{2\,\dot{\alpha}} \tilde{V}^b(\Pi_2) \tilde{V}^c(\Pi_3) \\
&\qquad \times \Big[\mathrm{Tr}(T^a [T^b, T^c]) \cosh(\pi_1 \wedge \pi_2) + \mathrm{Tr}(T^a \{T^b, T^c\}) \sinh(\pi_1 \wedge \pi_2) \Big]
\end{aligned} \tag{3.8}$$

where we use the convention that all the momenta are incoming.

We now consider the gauge–matter vertices. Their structures can be read from the effective actions (2.36) and (2.41). Performing FT of the terms which eventually contribute in a nontrivial way, for the cubic vertices in the chiral sector we find

$$\tilde{S}_1 = \frac{i}{4} \int d^8\Pi_1 d^8\Pi_2 d^8\Pi_3 \; \delta^{(8)}(\Pi_1 + \Pi_2 + \Pi_3) \; \overline{\pi}_2^2 \; \widetilde{\Gamma}^\alpha_a(\Pi_1)$$

$$\times \quad [(-i\pi_{2\alpha} - i\overline{\eth}_2^{\dot\alpha} p_{2\,\alpha\dot\alpha})\psi_b(\Pi_2)\overline{\psi}_c(\Pi_3) - \psi_b(\Pi_2)(-i\pi_{3\alpha} - i\overline{\eth}_3^{\dot\alpha} p_{3\,\alpha\dot\alpha})\overline{\psi}_c(\Pi_3)]$$
$$\times \quad \left\{\mathrm{Tr}(T^a[T^b,T^c])\cosh(\pi_1\wedge\pi_2) + \mathrm{Tr}(T^a\{T^b,T^c\})\sinh(\pi_1\wedge\pi_2)\right\} \tag{3.9}$$

$$\tilde{S}_2 = \frac{1}{4}\int d^8\Pi_1 d^8\Pi_2 d^8\Pi_3\ \delta^{(8)}(\Pi_1+\Pi_2+\Pi_3)\ \widetilde{\Gamma}_a^{\mu}(\Pi_1)\ [-p_{2\,\mu}+p_{3\,\mu}]\psi_b(\Pi_2)\overline{\psi}_c(\Pi_3)$$
$$\times \quad \left\{\mathrm{Tr}(T^a[T^b,T^c])\cosh(\pi_1\wedge\pi_2) + \mathrm{Tr}(T^a\{T^b,T^c\})\sinh(\pi_1\wedge\pi_2)\right\} \tag{3.10}$$

$$\tilde{S}_2' = -\frac{i}{4}\int d^8\Pi_1 d^8\Pi_2 d^8\Pi_3\ \delta^{(8)}(\Pi_1+\Pi_2+\Pi_3)\ \widetilde{W}_a^{\alpha}(\Pi_1)$$
$$\times \quad [(-i\pi_{2\alpha} - i\overline{\eth}_2^{\dot\alpha} p_{2\,\alpha\dot\alpha})\psi_b(\Pi_2)\overline{\psi}_c(\Pi_3) - \psi_b(\Pi_2)(-i\pi_{3\alpha} - i\overline{\eth}_3^{\dot\alpha} p_{3\,\alpha\dot\alpha})\overline{\psi}_c(\Pi_3)]$$
$$\times \quad \left\{\mathrm{Tr}(T^a[T^b,T^c])\cosh(\pi_1\wedge\pi_2) + \mathrm{Tr}(T^a\{T^b,T^c\})\sinh(\pi_1\wedge\pi_2)\right\} \tag{3.11}$$

whereas the quartic vertices are given by

$$\tilde{S}_1' = \frac{1}{4}\int d^8\Pi_1 d^8\Pi_2 d^8\Pi_3 d^8\Pi_4\ \delta^{(8)}(\Pi_1+\Pi_2+\Pi_3+\Pi_4)\ \overline{\pi}_4^2$$
$$\times\ \overline{\psi}_a(\Pi_1)\ \widetilde{\Gamma}_b^{\alpha}(\Pi_2)\ \widetilde{\Gamma}_{c\ \alpha}(\Pi_3)\ \psi_d(\Pi_4)$$
$$\times\ \mathcal{P}^{abcd}(\pi_1\pi_2\pi_3\pi_4) \tag{3.12}$$

$$\tilde{S}_2'' = -\frac{1}{4}\int d^8\Pi_1 d^8\Pi_2 d^8\Pi_3 d^8\Pi_4\ \delta^{(8)}(\Pi_1+\Pi_2+\Pi_3+\Pi_4)$$
$$\times\ \overline{\psi}_a(\Pi_1)\left(\widetilde{W}_b^{\alpha}(\Pi_2)\ \widetilde{\Gamma}_{c\ \alpha}(\Pi_3) + \widetilde{\Gamma}_b^{\alpha}(\Pi_2)\ \widetilde{W}_{c\ \alpha}(\Pi_3)\right)\ \psi_d(\Pi_4)$$
$$\times\ \mathcal{P}^{abcd}(\pi_1\pi_2\pi_3\pi_4) \tag{3.13}$$

$$\tilde{S}_2''' = -\frac{1}{4}\int d^8\Pi_1 d^8\Pi_2 d^8\Pi_3 d^8\Pi_4\ \delta^{(8)}(\Pi_1+\Pi_2+\Pi_3+\Pi_4)$$
$$\times\ \overline{\psi}_a(\Pi_1)\ \widetilde{\Gamma}_b^{\alpha\dot\alpha}(\Pi_2)\ \widetilde{\Gamma}_{c\ \alpha\dot\alpha}(\Pi_3)\ \psi_d(\Pi_4)$$
$$\times\ \mathcal{P}^{abcd}(\pi_1\pi_2\pi_3\pi_4) \tag{3.14}$$

with

$$\begin{aligned}\mathcal{P}^{abcd}(\pi_1\pi_2\pi_3\pi_4) = \{ & \mathrm{Tr}([T^a,T^b][T^c,T^d])\ \cosh(\pi_1\wedge\pi_2)\ \cosh(\pi_3\wedge\pi_4) \\ & + \mathrm{Tr}(\{T^a,T^b\}[T^c,T^d])\ \sinh(\pi_1\wedge\pi_2)\ \cosh(\pi_3\wedge\pi_4) \\ & + \mathrm{Tr}([T^a,T^b]\{T^c,T^d\})\ \cosh(\pi_1\wedge\pi_2)\ \sinh(\pi_3\wedge\pi_4) \\ & + \mathrm{Tr}(\{T^a,T^b\}\{T^c,T^d\})\ \sinh(\pi_1\wedge\pi_2)\ \sinh(\pi_3\wedge\pi_4)\}\end{aligned}$$

For the antichiral vertices we have analogous expressions with obvious changes.

4 One–loop diagrams

In this section, using the techniques described above, we compute the one–loop divergent contributions to the gauge effective action. In the ordinary case, in background field

method only the two–point function with chiral loop is divergent [23, 22]. In the NAC case, instead, we find divergent contributions up to the 4–point function for the gauge field due to the nontrivial modifications to the D–algebra induced by the star product.

We list our results without details. These will be reported elsewhere [24]. All the divergences are expressed in terms of a tadpole integral $\mathcal{T}$ and a self–energy $\mathcal{S}$ which in dimensional regularization ($n = 4 - 2\epsilon$) are

$$\mathcal{T} \equiv \int d^4q \, \frac{1}{q^2 + m\overline{m}} = -\frac{m\overline{m}}{(4\pi)^2} \, \frac{1}{\epsilon} + \mathcal{O}(1) \tag{4.1}$$

$$\mathcal{S} \equiv \int d^4q \, \frac{1}{((q-p)^2 + m\overline{m})(q^2 + m\overline{m})} = \frac{1}{(4\pi)^2} \, \frac{1}{\epsilon} + \mathcal{O}(1) \tag{4.2}$$

Other one–loop divergent integrals are obtained in terms of $\mathcal{T}$ and $\mathcal{S}$ through the following identities

$$\int d^4q \, \frac{q_{\alpha\dot{\alpha}}}{((q-p)^2 + m\overline{m})(q^2 + m\overline{m})} = \frac{1}{2} p_{\alpha\dot{\alpha}} \, \mathcal{S} \tag{4.3}$$

$$\int d^4q \, \frac{q_{\alpha\dot{\alpha}} q_{\beta\dot{\beta}}}{((q-p)^2 + m\overline{m})(q^2 + m\overline{m})} = \frac{1}{3} C_{\alpha\beta} C_{\dot{\alpha}\dot{\beta}} \left[\mathcal{T} - \frac{1}{2}(p^2 + 4m\overline{m})\mathcal{S} \right] \; + \; \frac{1}{3} \frac{p_{\alpha\dot{\alpha}} p_{\beta\dot{\beta}}}{p^2} \left[\mathcal{T} + (p^2 + m\overline{m})\mathcal{S} \right] \tag{4.4}$$

$$\int d^4q \, \frac{q^2}{((q-p)^2 + m\overline{m})(q^2 + m\overline{m})} = \mathcal{T} - m\overline{m}\mathcal{S} \tag{4.5}$$

$$\int d^4q \frac{q_{\alpha\dot{\alpha}} q_{\beta\dot{\beta}}}{(q^2 + m\overline{m})((q+p)^2 + m\overline{m})((q+r)^2 + m\overline{m})} \sim \frac{1}{2} C_{\alpha\beta} C_{\dot{\alpha}\dot{\beta}} \, \mathcal{S} \tag{4.6}$$

$$\int d^4q \frac{q_{\alpha\dot{\alpha}} q_{\beta\dot{\beta}} q_{\gamma\dot{\gamma}} q_{\rho\dot{\rho}}}{(q^2 + m\overline{m})((q+p)^2 + m\overline{m})((q+r)^2 + m\overline{m})((q+s)^2 + m\overline{m})} \sim \frac{1}{6} \mathcal{S} (C_{\alpha\beta} C_{\dot{\alpha}\dot{\beta}} C_{\gamma\rho} C_{\dot{\gamma}\dot{\rho}} + C_{\alpha\gamma} C_{\dot{\alpha}\dot{\gamma}} C_{\beta\rho} C_{\dot{\beta}\dot{\rho}} + C_{\alpha\rho} C_{\dot{\alpha}\dot{\rho}} C_{\beta\gamma} C_{\dot{\beta}\dot{\gamma}}) \tag{4.7}$$

In the massless case ($m = \overline{m} = 0$) the tadpole $\mathcal{T}$ vanishes due to a complete cancellation between the UV and the IR divergence. Consequently, the results for the self–energy type integrals can be obtained from (4.2 - 4.7) by setting $\mathcal{T} \sim 0$ and $m = \overline{m} = 0$.

4.1 Two–point function

Divergent contributions to the two–point function are listed in Fig. 1.

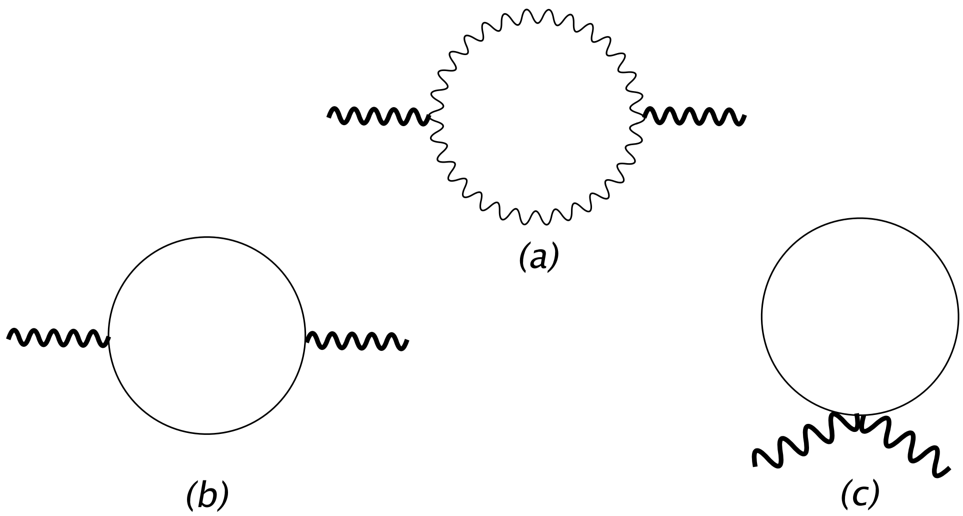

Figure 1: Gauge one–loop two–point functions.

Diagram (1a) with a vector loop can be computed by using Feynman rules (3.7) and (3.8). The divergent contribution turns out to be proportional to $\mathcal{F}^2$

$$-2\mathcal{F}^2 \int d^4x d^4\theta \, \mathrm{Tr}(\partial^2 W^{\dot\alpha})\mathrm{Tr}(W_{\dot\alpha}) = -2\mathcal{F}^2 \int d^4x d^4\theta \, \partial^2 \mathrm{Tr}(e_*^{-V} * \overline{W}^{\dot\alpha} * e_*^V)\mathrm{Tr}(e_*^{-V} * \overline{W}_{\dot\alpha} * e_*^V) \tag{4.8}$$

where we have expressed the covariantly antichiral field strength $W_{\dot\alpha}$ in terms of the ordinary antichiral one.
Given the particular group structure one can prove that this contribution is equal to

$$-2\mathcal{F}^2 \, \mathcal{S} \int d^4x d^4\theta \, (\partial^2 \mathrm{Tr}\overline{W}^{\dot\alpha}) \Big[\mathrm{Tr}\overline{W}_{\dot\alpha} + 2\mathcal{F}^{\alpha\beta}\mathrm{Tr}(\partial_\alpha e_*^V \partial_\beta (e_*^{-V} * \overline{W}_{\dot\alpha})) \Big] \tag{4.9}$$

and actually vanishes once integrated in $d^2\theta$.

We now consider matter loops (1b,1c) following the Feynman rules (3.7) and (3.9)–(3.14) for the chiral superfields and the analogous ones for the antichirals. This also covers contributions from ghosts up to an overall sign.
We focus on the chiral and the antichiral sectors separately.

• Chiral sector:
Order $\mathcal{F}^0$: This is the contribution which is already present in the the ordinary case. It is obtained by taking the π^2 factor inside the loop entirely from the covariant derivatives. Performing the explicit calculation we find

$$\mathcal{D}^{(2)} = \frac{1}{2}\mathcal{S} \int d^8z \Big[\mathcal{N} \, \mathrm{Tr}\,(\Gamma^\alpha \, W_\alpha) - \mathrm{Tr}\,(\Gamma^\alpha)\,\mathrm{Tr}\,(W_\alpha) \Big] \tag{4.10}$$

Order $\mathcal{F}$: Contributions proportional to a single power of $\mathcal{F}$ come from diagrams which have already a single power π from the covariant derivatives, whereas a second factor π

is produced by linearly expanding the hyperbolic functions. However, it is easy to realize that this expansion is always proportional to a trivial vanishing colour factor.

Order $\mathcal{F}^2$: These contributions are associated to nonplanar diagrams where no π factors come from covariant derivatives and the hyperbolic functions are expanded up to second order. After a bit of calculations we obtain

$$\begin{aligned}\frac{\mathcal{F}^2}{4}\int d^8z\,\Big\{&-\mathrm{Tr}\left((4\mathcal{T}-4m\overline{m}\mathcal{S}+\Box\,\mathcal{S})\partial^2\frac{D^2}{\Box}\Gamma^\alpha\right)\mathrm{Tr}(\overline{D}^2\Gamma_\alpha)\\ &+\frac{2}{3}i\mathrm{Tr}\left((2\mathcal{T}-4m\overline{m}\mathcal{S}+\Box\,\mathcal{S})\partial^2 D_\beta\frac{\partial^{\beta\dot{\alpha}}}{\Box}\Gamma^\alpha\right)\mathrm{Tr}(\overline{D}_{\dot{\alpha}}\Gamma_\alpha)\\ &+\frac{i}{3}\mathrm{Tr}\left((4\mathcal{T}+4m\overline{m}\mathcal{S}-\Box\,\mathcal{S})\frac{D^\alpha}{\Box}\,\partial^2\Gamma_\alpha\right)\mathrm{Tr}(\partial^{\beta\dot{\beta}}\overline{D}_{\dot{\beta}}\Gamma_\beta)\\ &-4\,\mathcal{T}\,\mathrm{Tr}(\partial^2\Gamma^\alpha)\,\mathrm{Tr}(\Gamma_\alpha)\Big\}\end{aligned} \tag{4.11}$$

It is possible to prove that this expression actually vanishes due to the particular structure of the star product hidden in Γ_α and the fact that in (4.11) only the $U(1)$ part of the connection appears.

- Antichiral sector:

Order $\mathcal{F}^0$: Also in this case this is the contribution which is already present in the ordinary case.

$$\overline{\mathcal{D}}^{(2)}=\frac{1}{2}\mathcal{S}\int d^8z\Big[\,\mathcal{N}\;\mathrm{Tr}\left(\overline{\Gamma}^{\dot{\alpha}}\;\overline{W}_{\dot{\alpha}}\right)-\mathrm{Tr}\left(\overline{\Gamma}^{\dot{\alpha}}\right)\mathrm{Tr}\left(\overline{W}_{\dot{\alpha}}\right)\Big] \tag{4.12}$$

Order $\mathcal{F}$: As in the chiral sector, this contribution is proportional to a trivial vanishing colour factor, then there is no contribution at this order.

Order $\mathcal{F}^2$: Divergent contributions proportional to $\mathcal{F}^2$ have the form

$$\mathcal{F}^2\int d^8z\overline{\theta}^2\;\Box\;\partial^2\overline{\Gamma}^{\dot{\alpha}}\;\overline{W}_{\dot{\alpha}}=\mathcal{F}^2\int d^8z\overline{\theta}^2\;\Box\;D^2\overline{\Gamma}^{\dot{\alpha}}\;\overline{W}_{\dot{\alpha}} \tag{4.13}$$

and trivially vanish when integrated in $d^2\theta$.

4.2 Three–point function

Divergent contributions to the three–point function are listed in Fig. 2.

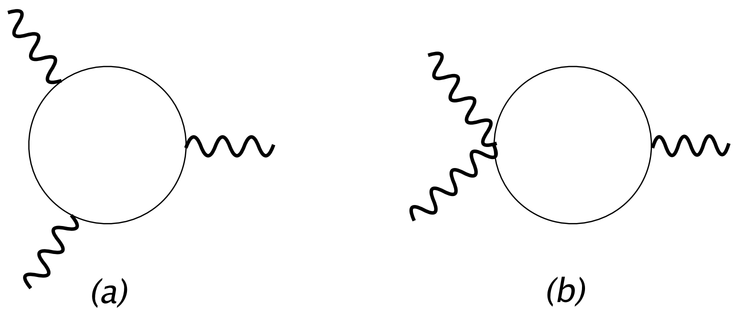

Figure 2: Gauge one–loop three–point functions.

Three-point diagrams with vector loop are finite, then we focus on matter loops. Both in the chiral and in the antichiral sectors, divergent terms are of order $\mathcal{F}$. Indeed, even if in principle we can have divergences also at order $\mathcal{F}^2$, the explicit calculation of the phase structures shows that these contributions cancel.

• Chiral sector:
We obtain

$$\begin{aligned}\mathcal{D}^{(3)} &= i\mathcal{S}\int d^4x d^2\theta\ \mathcal{F}^{\beta\gamma}[\mathrm{Tr}(\partial_\beta W^\alpha) * \mathrm{Tr}(\Gamma_\gamma * W_\alpha) - \mathrm{Tr}(\partial_\beta W^\alpha) * \mathrm{Tr}(W_\alpha * \Gamma_\gamma) \\ &\qquad - \mathrm{Tr}(\partial_\beta \Gamma_\gamma) * \mathrm{Tr}(W^\alpha * W_\alpha)]_{\overline{\theta}=0} \qquad (4.14)\end{aligned}$$

We have already performed the $\overline{\theta}$ integration since it allows for some cancellation among various terms. One can prove that the star products in (4.14) are actually ordinary products and the result can be rewritten as

$$\mathcal{D}^{(3)} = i\mathcal{S}\int d^4x d^2\theta\ \mathcal{F}^{\beta\gamma}[2\mathrm{Tr}(D_\beta W^\alpha)\mathrm{Tr}(\Gamma_\gamma W_\alpha) - \mathrm{Tr}(D_\beta\Gamma_\gamma)\mathrm{Tr}(W^\alpha W_\alpha)]_{\overline{\theta}=0} \qquad (4.15)$$

• Antichiral sector:
We obtain

$$\begin{aligned}\overline{\mathcal{D}}^{(3)} &= \frac{2}{3}\mathcal{S}\mathcal{F}^{\rho\gamma}\int d^8z\ \overline{\theta}^{\dot\beta}\Big[\mathrm{Tr}\left(\partial_\rho\overline{\Gamma}^{\dot\alpha}\right)\mathrm{Tr}\left(\overline{W}_{\dot\alpha}\overline{\Gamma}_{\gamma\dot\beta}\right) + \mathrm{Tr}\left(\partial_\rho\overline{W}^{\dot\alpha}\right)\mathrm{Tr}\left(\overline{\Gamma}_{\dot\alpha}\overline{\Gamma}_{\gamma\dot\beta}\right) \\ &\qquad + \mathrm{Tr}\left(\partial_\rho\overline{\Gamma}_{\gamma\dot\beta}\right)\mathrm{Tr}\left(\overline{W}^{\dot\alpha}\overline{\Gamma}_{\dot\alpha}\right)\Big] \qquad (4.16)\end{aligned}$$

In this case we perform both the θ and $\overline{\theta}$ integrations, in order to allow for some cancellations

$$\overline{\mathcal{D}}^{(3)} = i\mathcal{S}\mathcal{F}^{\rho\gamma}\int d^4x\Big[\ 2\ \partial_{\rho\dot\rho}\mathrm{Tr}\left(\overline{W}^{\dot\alpha}\right)\mathrm{Tr}\left(\overline{W}_{\dot\alpha}\overline{\Gamma}_\gamma^{\ \dot\rho}\right) + \partial_{\rho\dot\rho}\mathrm{Tr}\left(\overline{\Gamma}_\gamma^{\ \dot\rho}\right)\mathrm{Tr}\left(\overline{W}^{\dot\alpha}\overline{W}_{\dot\alpha}\right)\Big]_{\theta=\overline{\theta}=0} \qquad (4.17)$$

4.3 Four–point function

Divergent contributions to the four–point function are listed in Fig. 3.

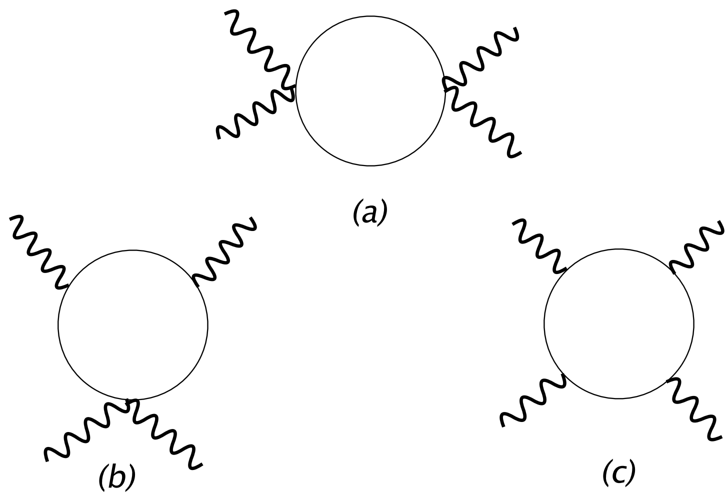

Figure 3: Gauge one–loop four–point functions.

Again, diagrams with vector loops give finite contributions. Considering matter loops we find divergences only at order $\mathcal{F}^2$.

• Chiral sector:

After $\overline{\theta}$ integration, we obtain

$$\begin{aligned}\mathcal{D}^{(4)} &= \mathcal{S}\mathcal{F}^2 \int d^4x d^2\theta \, \Big[\frac{1}{2}\partial^2 \mathrm{Tr}\left(\Gamma^\alpha * \Gamma_\alpha\right) \mathrm{Tr}\left(W^\beta * W_\beta\right) - \partial^2 \mathrm{Tr}\left(\Gamma^\alpha * W^\beta\right) \mathrm{Tr}\left(\Gamma_\alpha * W_\beta\right) \\ &\qquad - \mathrm{Tr}\left(\partial^2 \Gamma^\alpha\right) \mathrm{Tr}\left(\Gamma_\alpha * W^\beta * W_\beta\right) - \mathrm{Tr}\left(\partial^2 W^\alpha\right) \mathrm{Tr}\left(W_\alpha * \Gamma^\beta * \Gamma_\beta\right) \Big]_{\overline{\theta}=0}\end{aligned} \tag{4.18}$$

In this case it would be possible to replace all the star products with ordinary products in the first two terms, but not in the last two.

• Antichiral sector:

We obtain

$$\begin{aligned}\overline{\mathcal{D}}^{(4)} &= -\frac{1}{12}\mathcal{S}\mathcal{F}^2 \int d^8z \, \overline{\theta}^2 \Big[\partial^2 \mathrm{Tr}\left(\overline{W}^{\dot\alpha} * \overline{\Gamma}_{\dot\alpha}\right) \mathrm{Tr}\left(\overline{\Gamma}^{\gamma\dot\gamma} * \overline{\Gamma}_{\gamma\dot\gamma}\right) \\ &\qquad + 8 \, \partial^2 \mathrm{Tr}\left(\overline{\Gamma}^{\dot\alpha} * \overline{W}^{\dot\beta}\right) \mathrm{Tr}\left(\overline{W}_{\dot\beta} * \overline{\Gamma}_{\dot\alpha}\right) \Big] \\ &= -\frac{1}{2}\mathcal{S}\mathcal{F}^2 \int d^4x \Big[\mathrm{Tr}\left(\overline{W}^{\dot\alpha}\overline{W}_{\dot\alpha}\right) \mathrm{Tr}\left(\overline{W}^{\dot\beta}\overline{W}_{\dot\beta}\right)\Big]_{\theta=\overline{\theta}=0}\end{aligned} \tag{4.19}$$

5 (Super)gauge invariance

Collecting all the results of the previous section and performing the θ and $\overline{\theta}$ integrations for simplicity, the divergent part of the one–loop gauge effective action reads

$$
\begin{aligned}
\Gamma^{(1)}_{gauge} \;=\; & \frac{1}{2}(-3+N_f)\mathcal{S}\int d^4x \Big\{ \frac{1}{2}D^2\,\Big[\,\mathcal{N}\,\mathrm{Tr}\,(W^\alpha\,W_\alpha) - \mathrm{Tr}\,(W^\alpha)\,\mathrm{Tr}\,(W_\alpha)\Big] \\
& + \frac{1}{2}\overline{D}^2\,\Big[\,\mathcal{N}\,\mathrm{Tr}\left(\overline{W}^{\dot\alpha}\,\overline{W}_{\dot\alpha}\right) - \mathrm{Tr}\left(\overline{W}^{\dot\alpha}\right)\mathrm{Tr}\left(\overline{W}_{\dot\alpha}\right)\Big] \\
& + i\mathcal{F}^{\rho\gamma}D^2\,\left[2\mathrm{Tr}(D_\rho W^\alpha)\mathrm{Tr}(\Gamma_\gamma W_\alpha) - \mathrm{Tr}(D_\rho\Gamma_\gamma)\mathrm{Tr}(W^\alpha W_\alpha)\right] \\
& + i\mathcal{F}^{\rho\gamma}\Big[\,2\,\partial_{\rho\dot\rho}\mathrm{Tr}\left(\overline{W}^{\dot\alpha}\right)\mathrm{Tr}\left(\overline{W}_{\dot\alpha}\overline{\Gamma}_\gamma^{\ \dot\rho}\right) + \partial_{\rho\dot\rho}\mathrm{Tr}\left(\overline{\Gamma}_\gamma^{\ \dot\rho}\right)\mathrm{Tr}\left(\overline{W}^{\dot\alpha}\overline{W}_{\dot\alpha}\right)\Big] \\
& + \mathcal{F}^2\,\Big[\frac{1}{2}D^2\mathrm{Tr}\,(\Gamma^\alpha\Gamma_\alpha)\,D^2\mathrm{Tr}\left(W^\beta W_\beta\right) - D^2\mathrm{Tr}\left(\Gamma^\alpha W^\beta\right)D^2\mathrm{Tr}\,(\Gamma_\alpha W_\beta) \\
& \quad - \mathrm{Tr}\left(D^2\Gamma^\alpha\right)D^2\mathrm{Tr}\left(\Gamma_\alpha * W^\beta * W_\beta\right) - \mathrm{Tr}\left(D^2W^\alpha\right)D^2\mathrm{Tr}\left(W_\alpha * \Gamma^\beta * \Gamma_\beta\right)\Big] \\
& - \frac{1}{2}\mathcal{F}^2\Big[\mathrm{Tr}\left(\overline{W}^{\dot\alpha}\overline{W}_{\dot\alpha}\right)\mathrm{Tr}\left(\overline{W}^{\dot\beta}\overline{W}_{\dot\beta}\right)\Big]\Big\}_{\theta=\overline{\theta}=0} \\
\equiv\; & \frac{1}{2}(-3+N_f)[\mathcal{D}^{(2)} + \overline{\mathcal{D}}^{(2)} + \mathcal{D}^{(3)} + \overline{\mathcal{D}}^{(3)} + \mathcal{D}^{(4)} + \overline{\mathcal{D}}^{(4)}]
\end{aligned}
\tag{5.1}
$$

where factor $\frac{1}{2}$ comes from (2.32), (-3) is the contribution from the ghosts whereas N_f comes from matter. We note that the contributions to the two-point functions are independent of $\mathcal{F}$, three-point functions are linear in $\mathcal{F}$ and four-point functions are quadratic in $\mathcal{F}$.
We consider the variation of $\Gamma^{(1)}_{gauge}$ under supergauge transformation $e_*^{V'} = e_*^{i\overline{\Lambda}} * e_*^V * e_*^{-i\Lambda}$. Superfield strengths and superconnections transform as

$$
\begin{gathered}
\delta\Gamma_\alpha = D_\alpha\Lambda + i[\Lambda,\Gamma_\alpha]_* \qquad\qquad \delta W_\alpha = i[\Lambda, W_\alpha]_* \\
\delta\overline{\Gamma}_{\dot\beta} = \overline{D}_{\dot\beta}\overline{\Lambda} + i[\overline{\Lambda},\overline{\Gamma}_{\dot\beta}]_* \qquad \delta\overline{\Gamma}_{\beta\dot\beta} = \partial_{\beta\dot\beta}\overline{\Lambda} + i[\overline{\Lambda},\overline{\Gamma}_{\beta\dot\beta}]_* \qquad \delta\overline{W}_{\dot\beta} = i[\overline{\Lambda},\overline{W}_{\dot\beta}]_*
\end{gathered}
\tag{5.2}
$$

from which we can easily infer the transformation rules of the components appearing in (5.1). By expanding the $*$-product, after a long but straightforward calculation, it is possible to show that

$$
\delta\mathcal{D}^{(2)} = A\ \mathcal{S} \qquad\qquad \delta\mathcal{D}^{(3)} = -\left(A+B\right)\ \mathcal{S} \qquad\qquad \delta\mathcal{D}^{(4)} = B\ \mathcal{S} \tag{5.3}
$$

with

$$
\begin{aligned}
A \;=\; & 2i\mathcal{F}^{\rho\gamma}\int d^4x\Big[\mathrm{Tr}\left(D_\gamma\Lambda D_\rho W^\alpha\right)\mathrm{Tr}\left(D^2W_\alpha\right) - \mathrm{Tr}\left(D^2\Lambda D_\rho W^\alpha\right)\mathrm{Tr}\left(D_\gamma W_a\right) \\
& + \mathrm{Tr}\left(D_\gamma\Lambda D^2W^\alpha\right)\mathrm{Tr}\left(D_\rho W_\alpha\right)\Big]_{\theta=\overline{\theta}=0}
\end{aligned}
\tag{5.4}
$$

$$
\begin{aligned}
B \;=\; & -\mathcal{F}^2 \int d^4x \Big[2\mathrm{Tr}\left(D^2\Lambda D_\beta W^\alpha\right) D^2\mathrm{Tr}\left(\Gamma^\beta W_\alpha\right) - 2\mathrm{Tr}\left(D_\beta\Lambda D^2 W^\alpha\right) D^2\mathrm{Tr}\left(\Gamma^\beta W_\alpha\right) \\
& \quad -\mathrm{Tr}\left(D^2\Lambda D_\beta\Gamma^\beta\right) D^2\mathrm{Tr}\left(W^\alpha W_\alpha\right) + \mathrm{Tr}\left(D_\beta\Lambda D^2\Gamma^\beta\right) D^2\mathrm{Tr}\left(W^\alpha W_\alpha\right) \\
& \quad +\mathrm{Tr}\left(D^2 W^\alpha\right)\mathrm{Tr}\left(\left\{D^\beta\Gamma^\gamma, D_\gamma\Lambda\right\} D_\beta W_\alpha\right) - \mathrm{Tr}\left(D^2 W^\alpha\right)\mathrm{Tr}\left(\Gamma^\gamma\left\{D^2\Lambda, D_\gamma W_\alpha\right\}\right) \\
& \quad +\mathrm{Tr}\left(D^2 W^\alpha\right)\mathrm{Tr}\left(\Gamma^\gamma\left[D_\gamma\Lambda, D^2 W_\alpha\right]\right) + \mathrm{Tr}\left(D^2 W^\alpha\right)\mathrm{Tr}\left(\left\{D^2\Lambda, D_\gamma\Gamma^\gamma\right\} W_\alpha\right) \\
& \quad +\mathrm{Tr}\left(D^2 W^\alpha\right)\mathrm{Tr}\left(\left[D^2\Gamma^\gamma, D_\gamma\Lambda\right] W_\alpha\right) + \mathrm{Tr}\left(D^2\Gamma^\gamma\right)\mathrm{Tr}\left(D_\gamma\Lambda D^2 W^\alpha W_\alpha\right) \\
& \quad +\mathrm{Tr}\left(D^2\Gamma^\gamma\right)\mathrm{Tr}\left(D_\gamma\Lambda W^\alpha D^2 W_\alpha\right) + \mathrm{Tr}\left(D^2\Gamma^\gamma\right)\mathrm{Tr}\left(D_\gamma\Lambda D_\beta W^\alpha D^\beta W_\alpha\right) \\
& \quad -\mathrm{Tr}\left(D^2\Gamma^\gamma\right)\mathrm{Tr}\left(D^2\Lambda D_\gamma W^\alpha W_\alpha\right) + \mathrm{Tr}\left(D^2\Gamma^\gamma\right)\mathrm{Tr}\left(D^2\Lambda W^\alpha D_\gamma W_\alpha\right) \Big]_{\theta=\overline{\theta}=0} \\
& -\mathcal{F}^2\mathcal{F}^{\rho\gamma} \int d^4x \Big[\;\mathrm{Tr}\left(D^2 W^\alpha\right)\mathrm{Tr}\left(D_\rho\Gamma_\gamma\left[D^2\Lambda, D^2 W_\alpha\right]\right) \\
& \quad +\mathrm{Tr}\left(D^2 W^\alpha\right)\mathrm{Tr}\left(\left[D^2\Gamma_\gamma, D^2\Lambda\right] D_\rho W_\alpha\right) + \mathrm{Tr}\left(D^2\Gamma_\rho\right)\mathrm{Tr}\left(D^2\Lambda D^2 W^\alpha D_\gamma W_\alpha\right) \\
& \quad +\mathrm{Tr}\left(D^2\Gamma_\rho\right)\mathrm{Tr}\left(D^2\Lambda D_\gamma W^\alpha D^2 W_\alpha\right) \Big]_{\theta=\overline{\theta}=0}
\end{aligned}
\tag{5.5}
$$

whereas

$$
\begin{aligned}
\delta\overline{\mathcal{D}}^{(2)} &= -\delta\overline{\mathcal{D}}^{(3)} \\
&= -2i\mathcal{S}\mathcal{F}^{\rho\gamma} \int d^4x \;\mathrm{Tr}\left(\partial_{\rho\dot\rho}\overline{W}^{\dot\alpha}\right)\;\mathrm{Tr}\left(\partial_\gamma{}^{\dot\rho}\overline{\Lambda}\;\overline{W}_{\dot\alpha}\right)\Big|_{\theta=\overline{\theta}=0}
\end{aligned}
\tag{5.6}
$$

$$
\delta\overline{\mathcal{D}}^{(4)} = 0 \tag{5.7}
$$

We note that in the chiral sector the gauge variation, when evaluated in components, is proportional to $D_\gamma\Lambda|$ and $D^2\Lambda|$ but not to $\Lambda|$. Therefore, in this sector ordinary gauge invariance is preserved term by term, whereas the supergauge one seems to be broken. In the antichiral sector instead, the gauge variation of each term is proportional to $\overline{\Lambda}|$ so breaking ordinary gauge invariance. However it is easy to see that all the variations sum up to zero and we find

$$
\delta\Gamma^{(1)}_{gauge} = 0 \tag{5.8}
$$

We have then proved the supergauge invariance of the one–loop gauge effective action.

6 Conclusions

In this paper we have computed one–loop divergent contributions to the gauge effective action for N=1/2 $U(\mathcal{N})$ SYM theory with matter in the adjoint representation of the gauge group. It turns out that new divergent terms proportional to the NAC parameter appear for the three and four–point functions of the gauge field. Term by term these corrections are not (super)gauge invariant being proportional not only to superfield–strengths but also to superconnections. However, we have proved that the complete divergent part of the effective action at one–loop is *supergauge invariant* since nontrivial cancellations

occur among the gauge variations of the two and three point functions and of the three and four point functions. This allows us to conclude that at least at one–loop there must be super Ward identities at work which guarantee the invariance of the theory, despite the appearance of a NAC product which breaks explicitly supersymmetry and is not invariant under supergauge transformations. Therefore, a pattern similar to the one studied for SYM theories with bosonic noncommutativity [20] seems to be present also in this case.

The analysis has been carried on by using a manifestly gauge invariant superspace setup. This has been accomplished by working directly with the star product (no expansions at the level of the action), performing Fourier transform also on the spinorial coordinates so trading the star products for spinorial phases and adapting the D–algebra in order to reduce the supergraphs to ordinary momentum integrals.

In order to study the supergauge invariance of the result we have found convenient to generalize the background field method to the NAC theory. In this approach the contributions to the effective action turn out to be proportional to the geometric objects of the theory, i.e. superconnections and superfield–strengths. The generalization of the background field method to the NAC case has revealed not straightforward for two main reasons: The hermitian conjugation rules in the classical action change in euclidean signature and some important identities involving covariant derivatives do not hold anymore because of the presence of a noncommutative product.

Our approach is particularly useful when the relation between supergauge invariance and NAC geometry at higher loops is of concern. Moreover, it makes possible the evaluation of higher–loop corrections to the effective action and higher–loop correlation functions for composite operators which in general is quite prohibitive in components. In principle, our method could be also generalized to the study of SYM theories with extended supersymmetry not broken [28] or partially broken by nonanticommutativity [29].

In this paper we have focused on the supergauge invariance of the gauge part of the effective action. Divergent contributions with matter on the external lines have still to be computed. Moreover, renormalizability of NAC SYM theories in superspace and the relation of our results with the ones found in components [15, 16] have not been discussed yet. This all will be the subject of a future publication [24].

Acknowledgments We thank M.T. Grisaru for his fundamental contribution at the early stage of the work. This work has been supported in part by INFN, COFIN prot. 2003023852_008 and the European Commission RTN program MRTN–CT–2004–005104.

References

[1] S. Ferrara and M. A. Lledo, “Some aspects of deformations of supersymmetric field theories”, JHEP **0005** (2000) 008, hep-th/0002084
S. Ferrara, M. A. Lledó and O. Maciá, “Supersymmetry in noncommutative superspaces”, JHEP **0309** (2003) 068, hep-th/0307039.

[2] D. Klemm, S. Penati and L. Tamassia, “Non(anti)commutative superspace”, Class.Quant.Grav. 20 (2003) 2905-2916, hep-th/0104190.

[3] J. de Boer, P.A. Grassi and P. van Nieuwenhuizen, “Non-commutative superspace from string theory”, Phys. Lett. B **574** (2003) 98, hep-th/0302078.

[4] H. Ooguri and C. Vafa, “The C–Deformation of Gluino and Non–planar Diagrams”, Adv.Theor.Math.Phys.7:53-85,2003, hep-th/0302109; “Gravity Induced C–Deformation”, Adv.Theor.Math.Phys.7:405-417,2004, hep-th/0303063.

[5] N. Seiberg, “Noncommutative Superspace, $N = \frac{1}{2}$ Supersymmetry, Field Theory and String Theory”, JHEP **0306** (2003) 010, hep-th/0305248.

[6] N. Berkovits and N. Seiberg, “Superstring in Graviphoton Background and $N = \frac{1}{2} + \frac{3}{2}$ Supersymmetry”, JHEP **0307** (2003) 010, hep-th/0306226.

[7] E. Ivanov, O. Lechtenfeld and B. Zupnik, “Nilpotent deformations of N = 2 superspace”, JHEP **0402** (2004) 012, hep-th/0308012.

[8] M.T. Grisaru, S. Penati and A. Romagnoni, “Two-loop renormalization for nonanticommutative N = 1/2 supersymmetric WZ model”, JHEP **0308** (2003) 003, hep-th/0307099.

[9] M. T. Grisaru, S. Penati and A. Romagnoni, “Nonanticommutative superspace and N = 1/2 WZ model”, Class. Quant. Grav. **21** (2004) S1391, hep-th/0401174

[10] R. Britto and B. Feng, “N = 1/2 Wess-Zumino model is renormalizable”, Phys. Rev. Lett. **91** (2003) 201601, hep-th/0307165.

[11] A. Romagnoni, “Renormalizability of N = 1/2 Wess-Zumino model in superspace”, JHEP **0310** (2003) 016, hep-th/0307209.

[12] R. Britto, B. Feng and S. J. Rey, “Deformed superspace, N = 1/2 supersymmetry and (non)renormalization theorems”, JHEP **0307** (2003) 067, hep-th/0306215.

[13] S. Terashima and J. T. Yee, “Comments on noncommutative superspace”, JHEP **0312** (2003) 053, hep-th/0306237.

[14] O. Lunin and S. J. Rey, “Renormalizability of non(anti)commutative gauge theories with N = 1/2 supersymmetry”, JHEP **0309** (2003) 045, hep-th/0307275;
D. Berenstein and S. J. Rey, “Wilsonian proof for renormalizability of N = 1/2 supersymmetric field theories”, Phys. Rev. D **68** (2003) 121701, hep-th/0308049.

[15] M. Alishahiha, A. Ghodsi and N. Sadooghi, “One-loop perturbative corrections to non(anti)commutativity parameter of N = 1/2 supersymmetric U(N) gauge theory”, Nucl. Phys. B **691** (2004) 111, hep-th/0309037.

[16] I. Jack, D. R. T. Jones and L. A. Worthy, “One-loop renormalisation of N=1/2 supersymmetric gauge theory”, hep-th/0412009.

[17] T. Araki, K. Ito and A. Ohtsuka, “Supersymmetric gauge theories on noncommutative superspace”, Phys. Lett. B **573** (2003) 209, hep-th/0307076.

[18] H. Liu and J. Michelson “*-Trek: The one-loop N = 4 noncommutative SYM action”, Nucl. Phys. B **614** (2001) 279, hep-th/0008205;
T. Mehen and M. B. Wise,“Generalized *-products, Wilson lines and the solution of the Seiberg-Witten equations”, JHEP **0012** (2000) 008, hep-th/0010204.

[19] A. Santambrogio and D. Zanon, “One-loop four-point function in noncommutative N = 4 Yang-Mills theory”, JHEP **0101** (2001) 024, hep-th/0010275;
D. Zanon,“Noncommutative N = 1,2 super U(N) Yang-Mills: UV/IR mixing and effective action results at one loop”, Phys. Lett. B **502** (2001) 265, hep-th/0012009.

[20] M. Pernici, A. Santambrogio and D. Zanon, “The one-loop effective action of noncommutative N = 4 super Yang-Mills is gauge invariant”, Phys. Lett. B **504** (2001) 131, hep-th/0011140.

[21] R. Britto, B. Feng and S. J. Rey, “Non(anti)commutative superspace, UV/IR mixing and open Wilson lines”, JHEP **0308** (2003) 001, hep-th/0307091.

[22] S.J. Gates, Jr., M.T. Grisaru, M. Roček and W. Siegel, *Superspace*, Benjamin Cummings, (1983) Reading, MA.

[23] M. T. Grisaru, W. Siegel and M. Rocek, “Improved Methods For Supergraphs”, Nucl. Phys. B **159** (1979) 429.
M. T. Grisaru and W. Siegel, “Supergraphity. 2. Manifestly Covariant Rules And Higher Loop Finiteness”, Nucl. Phys. B **201** (1982) 292
M. T. Grisaru and D. Zanon, “Covariant Supergraphs. 1. Yang-Mills Theory”, Nucl. Phys. B **252** (1985) 578.

[24] M.T. Grisaru, S. Penati and A. Romagnoni, in preparation.

[25] T. Filk, “Divergencies in a field theory on quantum space”, Phys. Lett. B **376** (1996) 53.

[26] S. Minwalla, M. Van Raamsdonk and N. Seiberg, "Noncommutative perturbative dynamics", JHEP **0002** (2000) 020, hep-th/9912072.

[27] K. G. Chetyrkin, A. L. Kataev and F. V. Tkachov, "New Approach To Evaluation Of Multiloop Feynman Integrals: The Gegenbauer Polynomial X Space Technique", Nucl. Phys. B **174** (1980) 345.
K. G. Chetyrkin and F. V. Tkachov, "Integration By Parts: The Algorithm To Calculate Beta Functions In 4 Loops", Nucl. Phys. B **192** (1981) 159

[28] S. Ferrara and E. Sokatchev, "Non-anticommutative N = 2 super-Yang-Mills theory with singlet deformation", Phys. Lett. B **579** (2004) 226, hep-th/0308021.

[29] S. Ferrara, E. Ivanov, O. Lechtenfeld, E. Sokatchev and B. Zupnik, "Non-anticommutative chiral singlet deformation of N = (1,1) gauge theory", hep-th/0405049.